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General Instructions:

1) This is a **closed-book** exam. Calculators **are** permitted, but **sharing** of calculators or any other material between students is **not** permitted.
2) You are allowed a single 8.5" X 11" sheet of paper which may contain handwriting on both sides. Photocopied sheets are not allowed.
3) This is a multiple-choice exam: **circle the letter** of the appropriate answer(s) for each question.
4) Questions may have **no right answer**, **one right answer**, or **more than one right answer**.
5) The total number of right answers on this exam is 42. **Do not** circle more than this number of answers. One mark will be subtracted for each circled answer in excess of 42. For example if you circle 50 answers then 8 marks will be subtracted.
6) **Use the back of each page to do any rough calculations. If you require more scrap paper ask and I will give you some.**
7) Make sure that your name, student number, and signature are written on this page.

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TOTAL: **/ 42**

1. Consider the 1-D Maxwell's equations written in conservation-law form:

$$\partial_t \boldsymbol{u}(x, t) + A \partial_x \boldsymbol{u}(x, t) = \mathbf{0}$$

where the solution vector $\boldsymbol{u}$ is given by

$$\boldsymbol{u} = \begin{bmatrix} E_y \\ H_z \end{bmatrix}, \text{ and } A = \begin{bmatrix} 0 & e \\ m & 0 \end{bmatrix}, \; e = \frac{1}{\varepsilon}, \; m = \frac{1}{\mu}, \; C^2 = me.$$

Numerical dispersion occurs in the Yee algorithm solution of this system

a) if we choose $\Delta t > C\Delta x$

b) if we choose $\Delta t < C\Delta x$

c) if we choose $\Delta t = C\Delta x$

d) for all choices of Δt

e) only if we have less than 10 spatial cells per wavelength

f) none of the above.

2. The *Successive Over-Relaxation* method

a) is a method of discretizing Laplace's equation

b) is always stable

c) is a method of solving only sparse matrix equations

d) is a method of solving any matrix equation

e) is only stable for a value of the relaxation parameter ω chosen greater than 2.

f) none of the above.

3. The difference equation:

$$\frac{u_i^n - u_{i-1}^n}{\Delta x}$$

for the grid function given by $u_i^n \cong u(i\Delta x, n\Delta t)$ is:

a) a second-order accurate approximation to the derivative of $u(x, t)$ with respect to space at time $n\Delta t$ and spatial point $(i + 1/2)\Delta x$

b) a first-order accurate approximation to the derivative of $u(x, t)$ with respect to space at time $n\Delta t$ and spatial point $i\Delta x$

c) a first-order accurate approximation to the derivative of $u(x, t)$ with respect to space at time $n\Delta t$ and spatial point $(i - 1)\Delta x$

d) a first-order approximation to the second derivative of $u(x, t)$ with respect to space at time $n\Delta t$ and spatial point $(i + 1/2)\Delta x$

e) none of the above.

4. The difference equation: $\dfrac{u_{i-1,j} + u_{i+1,j} - 2u_{i,j}}{(\Delta x)^2} + \dfrac{u_{i,j-1} + u_{i,j+1} - 2u_{i,j}}{(\Delta y)^2} = f_{i,j}$

for the grid functions given by $u_{i,j} \cong u(i\Delta x, j\Delta y)$ and $f_{i,j} \cong f(i\Delta x, j\Delta y)$ is:

a) a second-order accurate approximation to Laplace's equation: $\nabla^2 u(x, y) = 0$

b) a first-order accurate approximation to Laplace's equation: $\nabla^2 u(x, y) = 0$

c) a second-order accurate approximation to Poisson's equation: $\nabla^2 u(x, y) = -f(x, y)$

d) a second-order accurate approximation to Poisson's equation: $\nabla^2 u(x, y) = f(x, y)$

e) none of the above.

5. If we apply the finite element method to the problem defined as:

$$\frac{d^2}{dt^2}u(t) = 2,\ u(0) = 0,\ u(1) = 1,\ \text{on the interval } 0 \le t \le 1$$

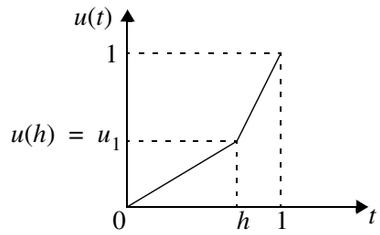
using *linear* basis functions, $\alpha(t) = at + b$, on the two elements defined by $0 \le t \le h$ and $h \le t \le 1$ then the approximate value of $u(h) = u_1$ we will obtain is

a) $u_1 = h$ b) $u_1 = 1/2$

c) $u_1 = h^2$ d) $u_1 = (1-h)/2$

e) $u_1 = (1-h)^2$

f) none of the above.



6. The notation $u'(i\Delta t) = F(u^{n-1}, u^n, u^{n+1}, \Delta t) + O((\Delta t)^3)$ where $F(u_{i-1}, u_i, u_{i+1}, \Delta t)$ is a difference formula, means that

a) $F(u^{n-1}, u^n, u^{n+1}, \Delta t)$ can always be effectively used to approximate the derivative $u'(i\Delta t)$ in the finite difference solution of a differential equation.

b) as the time-step Δt goes to zero the term $O((\Delta t)^3)$ goes to zero.

c) $F(u^{n-1}, u^n, u^{n+1}, \Delta t)$ goes to zero faster than $(\Delta t)^2$ goes to zero.

d) $O((\Delta t)^3)/(\Delta t)^3 = K$ with finite K for any Δt.

e) if we do our calculations with at least three significant digits then we will have an exact approximation of the derivative.

7. Assume we have been given a finite difference approximation of a partial differential equation for the dependent variable $u(x, t)$ and have been told that it is of order $O(\Delta x)O((\Delta t)^2)$. The grid function is denoted $u_i^n \cong u(i\Delta x, n\Delta t)$ and the grid spacings Δx and Δt are not functions of position (i.e., we have a uniform grid). Which of the following are true?

a) If we reduce Δx by a factor of two then the error in the grid function will go down by a factor of two.

b) If we reduce Δx by a factor of two then the error in the grid function will go down by a factor of four.

c) If we reduce Δt by a factor of two then the error in the grid function will go down by a factor of two.

d) If we reduce Δt by a factor of two then the error in the grid function will go down by a factor of four.

e) If we reduce both Δx and Δt each by a factor of two then the error in the grid function will go down by a factor of four.

8. Assume we have been given a finite difference approximation of a partial differential equation for the dependent variable $u(x, t)$ and have been told that it is of order $O(\Delta x) + O((\Delta t)^2)$. The grid function is denoted $u_i^n \cong u(i\Delta x, n\Delta t)$ and the grid spacings Δx and Δt are not functions of position (i.e., we have a uniform grid). Which of the following are true?

a) If we reduce Δx by a factor of two then the error in the grid function will go down by a factor of two.

b) If we reduce Δx by a factor of two then the error in the grid function will go down by a factor of four.

c) If we reduce Δt by a factor of two then the error in the grid function will go down by a factor of two.

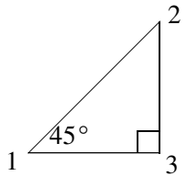
d) If we reduce Δt by a factor of two then the error in the grid function will go down by a factor of four.

e) If we reduce both Δx and Δt each by a factor of two then the error in the grid function will go down by a factor of four.

9. If we use Simpson's rule to integrate $u(x) = x^3 - x^2 + 2x$ over the interval $0 \le x \le 1$ using 4 intervals the answer will be:

a) 5/6 b) 11/12 c) 0.15 d) 0.8 e) 0.7604

10. Given the right-angle equilateral triangular element of area A with node numbers as shown, the first-order elemental stiffness matrix associated with it when using the finite element method to solve the Laplace equation is given by



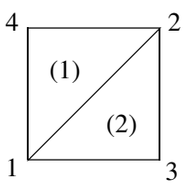
a) $S^{(e)} = \begin{bmatrix} 0.5 & -0.5 & -0.5 \\ -0.5 & 0.5 & 0 \\ -0.5 & 0 & 1.0 \end{bmatrix}$ b) $S^{(e)} = \frac{1}{A}\begin{bmatrix} 0.5 & 0 & -0.5 \\ 0 & 0.5 & -0.5 \\ -0.5 & -0.5 & 1 \end{bmatrix}$

c) $S^{(e)} = \begin{bmatrix} 0.5 & 0 & -0.5 \\ 0 & 0.5 & -0.5 \\ -0.5 & -0.5 & 1 \end{bmatrix}$ c) $S^{(e)} = \frac{1}{A}\begin{bmatrix} 1.0 & 0 & -0.5 \\ 0 & 0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 \end{bmatrix}$ d) $S^{(e)} = \begin{bmatrix} 1.0 & 0 & -0.5 \\ 0 & 0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 \end{bmatrix}$

e) $S^{(e)} = \frac{1}{A}\begin{bmatrix} 0.5 & -0.5 & -0.5 \\ -0.5 & 0.5 & 0 \\ -0.5 & 0 & 1.0 \end{bmatrix}$

f) none of the above

11. Given the two triangular elements with global node numbers as shown, and first-order elemental stiffness matrices given as



$$S^{(1)} = \begin{bmatrix} s_{11}^{(1)} & s_{12}^{(1)} & s_{13}^{(1)} \\ s_{21}^{(1)} & s_{22}^{(1)} & s_{23}^{(1)} \\ s_{31}^{(1)} & s_{32}^{(1)} & s_{33}^{(1)} \end{bmatrix} \text{ and } S^{(2)} = \begin{bmatrix} s_{11}^{(2)} & s_{12}^{(2)} & s_{13}^{(2)} \\ s_{21}^{(2)} & s_{22}^{(2)} & s_{23}^{(2)} \\ s_{31}^{(2)} & s_{32}^{(2)} & s_{33}^{(2)} \end{bmatrix}$$

where the local numbering is counter-clockwise starting at the left-most lower node of each triangle, these will contribute to the global stiffness matrix as

a) $S = \begin{bmatrix} s_{11}^{(1)} & s_{12}^{(1)} & s_{13}^{(1)} & 0 & 0 & 0 & \dots \\ s_{21}^{(1)} & s_{22}^{(1)} & s_{23}^{(1)} & 0 & 0 & 0 & \dots \\ s_{31}^{(1)} & s_{32}^{(1)} & s_{33}^{(1)} & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & s_{11}^{(2)} & s_{12}^{(2)} & s_{13}^{(2)} & \dots \\ 0 & 0 & 0 & s_{21}^{(2)} & s_{22}^{(2)} & s_{23}^{(2)} & \dots \\ 0 & 0 & 0 & s_{31}^{(2)} & s_{32}^{(2)} & s_{33}^{(2)} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots \end{bmatrix}$ b) $S = \begin{bmatrix} s_{11}^{(2)} & s_{12}^{(2)} & s_{13}^{(2)} & 0 & 0 & 0 & \dots \\ s_{21}^{(2)} & s_{22}^{(2)} & s_{23}^{(2)} & 0 & 0 & 0 & \dots \\ s_{31}^{(2)} & s_{32}^{(2)} & s_{33}^{(2)} & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & s_{11}^{(1)} & s_{12}^{(1)} & s_{13}^{(1)} & \dots \\ 0 & 0 & 0 & s_{21}^{(1)} & s_{22}^{(1)} & s_{23}^{(1)} & \dots \\ 0 & 0 & 0 & s_{31}^{(1)} & s_{32}^{(1)} & s_{33}^{(1)} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots \end{bmatrix}$

c) $S = \begin{bmatrix} s_{11}^{(1)} + s_{11}^{(2)} & s_{12}^{(1)} + s_{13}^{(2)} & s_{12}^{(2)} & s_{13}^{(1)} & 0 & 0 & \dots \\ s_{21}^{(1)} + s_{31}^{(2)} & s_{22}^{(1)} + s_{33}^{(2)} & s_{32}^{(2)} & s_{23}^{(1)} & 0 & 0 & \dots \\ s_{21}^{(2)} & s_{23}^{(2)} & s_{22}^{(2)} & 0 & 0 & 0 & \dots \\ s_{31}^{(1)} & s_{32}^{(1)} & 0 & s_{33}^{(1)} & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots \end{bmatrix}$ d) $S = \begin{bmatrix} s_{11}^{(1)} + s_{11}^{(2)} & s_{21}^{(1)} + s_{31}^{(2)} & s_{21}^{(2)} & s_{31}^{(1)} & 0 & 0 & \dots \\ s_{12}^{(1)} + s_{13}^{(2)} & s_{22}^{(1)} + s_{33}^{(2)} & s_{23}^{(2)} & s_{32}^{(1)} & 0 & 0 & \dots \\ s_{12}^{(2)} & s_{32}^{(2)} & s_{22}^{(2)} & 0 & 0 & 0 & \dots \\ s_{13}^{(1)} & s_{23}^{(1)} & 0 & s_{33}^{(1)} & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots \end{bmatrix}$

f) none of the above

12. In general, the finite element method cannot be used to solve non-linear electromagnetic field problems:

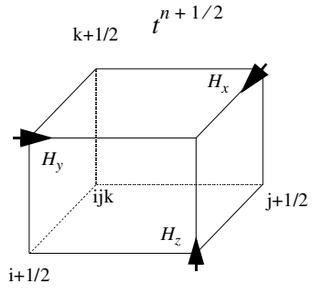
a) the above statement is **True** b) the above statement is **False**

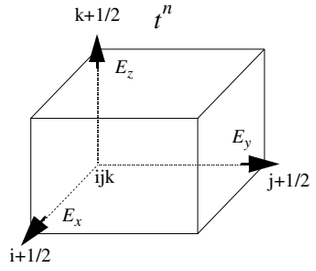
13. The second-order Mur absorbing boundary conditions:
    a) can only be used with the Yee FDTD algorithm
    b) are second order accurate in both space and time
    c) absorb any dispersion which occurs in FDTD
    d) can be used with both the scattered field formulation and the total field formulation of FDTD
    e) can be effectively used to truncate a finite element solution of the Laplace equation
    f) none of the above

14. The second-order Mur absorbing boundary conditions:
    a) cannot be used at the corners of a three-dimensional grid
    b) can only be used for three-dimensional FDTD solutions of Maxwell's equations
    c) completely absorbs electromagnetic plane waves incident from any angle
    d) none of the above

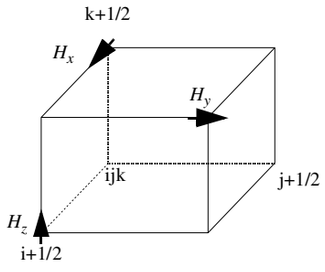
15. Which of the following are examples of valid FDTD grids:

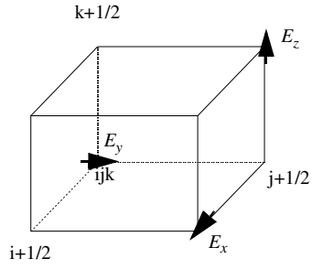
a)



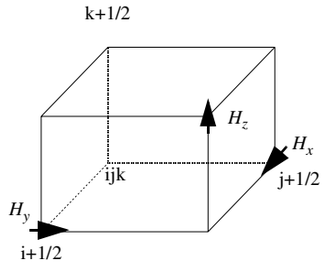


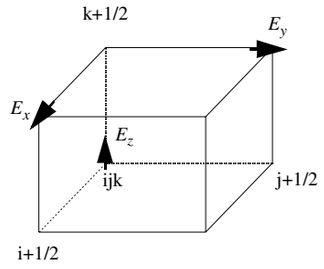
b)



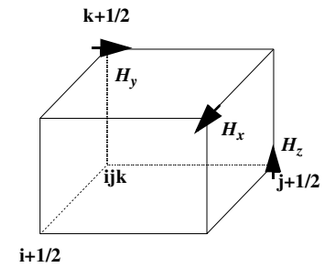


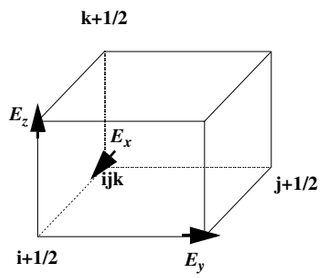
c)





d)





16. Consider the linear operator equation

$$Lf(\boldsymbol{x}) = g(\boldsymbol{x}),\ \boldsymbol{x} \in \mathbb{R}^3$$

where $g(\boldsymbol{x})$ is given and we are asked to find an approximation to $f(\boldsymbol{x})$. We can begin to solve this problem as follows. We first expand $f(\boldsymbol{x})$ as a sum of basis functions, $f_n(\boldsymbol{x})$:

$$f = \sum_{n=1}^{N} a_n f_n$$

where we now need to determine the coefficients a_n. We can then substitute this expansion into the operator equation to get

$$Lf = L\left(\sum_{n=1}^{N} a_n f_n\right) = \sum_{n=1}^{N} a_n L(f_n) = g(\boldsymbol{x}). \qquad (1)$$

This gives us one equation in N unknown coefficients. Which of the following statements are **true**:

a) If we take the inner product of equation (1) with any N arbitrary weighting functions we can always find a solution for the coefficients a_n.

b) If we take the inner product of equation (1) with N Dirac delta functions $\delta(\boldsymbol{x} - \boldsymbol{x}_n)$ with N unique values of $\boldsymbol{x}_n$, this is called *Point-Matching* or *Collocation*, and is a standard Method of Moments technique.

c) We can try to minimize the Euclidean norm of the residual

$$\Re = \sum_{n=1}^{N} a_n L(f_n) - g(\boldsymbol{x})$$

by varying the coefficients a_n as parameters in the minimization. This is equivalent to using the complex conjugate of $L(f_n)$ as weighting functions. That is $w_n^* = L(f_n)$.

d) The least-squares method is the same as Galerkin's Method.

e) If the basis functions are sub-domain (basis functions) then the weighting functions in the Method of Moments must also be sub-domain (weighting functions).

f) The Method of Moments can only be used with self-adjoint operators but the operator doesn't necessarily have to be positive definite.

g) If the operator is self-adjoint and positive definite then the Galerkin method is equivalent to the Finite Element technique as long as the basis functions are not in the null-space of the operator (i.e., as long as $L(f_n) \neq 0$ for arbitrary f_n).

h) Antenna problems cannot be solved using the Finite Element technique because it can only be used to solve differential equations and not integral equations.

i) If we use the Method of Moments to solve scattering problems formulated in terms of integral equations then when we change the angle of the incident plane wave we must re-calculate the matrix elements and re-invert the matrix.

j) The *Successive Over-Relaxation* method cannot be used with the Method of Moments.

k) Every Method of Moments solution of a partial differential equation problem has an equivalent finite difference formulation.

l) If the basis functions in the Method of Moments are sub-domain basis functions then the coefficients in the expansion

$$\sum_{n=1}^{N} a_n f_n$$

correspond to values of the unknown function at the node points which define each sub-domain.

m) Pocklington's Integro-differential equation cannot be solved using the Method of Moments and that is why we convert it to Hallén's integral equation.

n) The Method of Moments cannot be used on triangular grids in two-dimensions.

17. When we use the Rao-Wilton-Glisson elements for scattering problems, we define edge-based vector basis functions as follows (see Figure 1):

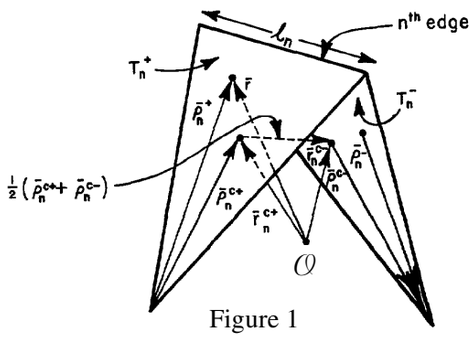


Figure 1

$$f_n(\boldsymbol{r}) = \begin{cases} \dfrac{l_n}{2A_n^+}\rho_n^+ & \boldsymbol{r} \text{ in } T_n^+ \\ \dfrac{l_n}{2A_n^-}\rho_n^- & \boldsymbol{r} \text{ in } T_n^- \\ 0 & \text{otherwise} \end{cases}$$

where ρ_n^+ and ρ_n^- are position vectors with respect to the free nodes of triangles T_n^+ and T_n^-, respectively. If for a particular triangulation of a flat plate lying in the *xy*-plane we have edge 1 associated with the two triangles as shown in Figure 2, then $\boldsymbol{f}_1(\boldsymbol{r})$ is given by:

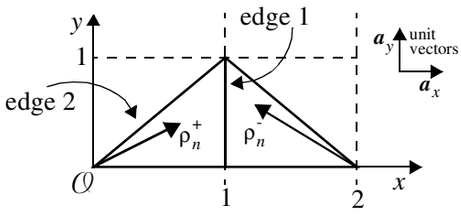


Figure 2

a) $$\boldsymbol{f}_1(x,y) = \frac{1}{2}\begin{cases} x\boldsymbol{a}_x + y\boldsymbol{a}_y & \boldsymbol{r} \text{ in } T_1^+ \\ -[(x-2)\boldsymbol{a}_x + y\boldsymbol{a}_y] & \boldsymbol{r} \text{ in } T_1^- \\ 0 & \text{otherwise} \end{cases}$$

b) $$\boldsymbol{f}_1(x,y) = \begin{cases} x\boldsymbol{a}_x + y\boldsymbol{a}_y & \boldsymbol{r} \text{ in } T_1^+ \\ -[(x-2)\boldsymbol{a}_x + y\boldsymbol{a}_y] & \boldsymbol{r} \text{ in } T_1^- \\ 0 & \text{otherwise} \end{cases}$$

c) $$\boldsymbol{f}_1(x,y) = \frac{1}{2}\begin{cases} x\boldsymbol{a}_x + y\boldsymbol{a}_y & \boldsymbol{r} \text{ in } T_1^+ \\ -[(x-2)\boldsymbol{a}_x - y\boldsymbol{a}_y] & \boldsymbol{r} \text{ in } T_1^- \\ 0 & \text{otherwise} \end{cases}$$

d) $$\boldsymbol{f}_1(x,y) = \begin{cases} x\boldsymbol{a}_x + y\boldsymbol{a}_y & \boldsymbol{r} \text{ in } T_1^+ \\ -[(x-1)\boldsymbol{a}_x - y\boldsymbol{a}_y] & \boldsymbol{r} \text{ in } T_1^- \\ 0 & \text{otherwise} \end{cases}$$

e) $$\boldsymbol{f}_1(x,y) = \frac{1}{2}\begin{cases} x\boldsymbol{a}_x + y\boldsymbol{a}_y & \boldsymbol{r} \text{ in } T_1^+ \\ -[(x-1)\boldsymbol{a}_x - y\boldsymbol{a}_y] & \boldsymbol{r} \text{ in } T_1^- \\ 0 & \text{otherwise} \end{cases}$$

f) none of the above.

18. The Rao-Wilton-Glisson vector basis functions given above are used to expand the surface current on the scattering object:

$$\boldsymbol{J}(\boldsymbol{r}) \cong \sum_{n=1}^{N} I_n \boldsymbol{f}_n(\boldsymbol{r}) .$$

The charge density on each triangle is given by $\sigma = -\nabla_s \cdot \boldsymbol{J}/(j\omega)$. For the triangulation shown in Figure 2 above, which of the following statements are true?

a) The total charge in triangle T_1^+ is the same as the total charge in triangle T_1^-.

b) The total charge in triangle T_1^+ has the same magnitude but opposite sign as the total charge in triangle T_1^-.

c) The total charge in each triangle is always zero.

d) The magnitude of the total charge in each triangle will be $|I_1/(j\omega)|$.

e) The magnitude of the total charge in each triangle will be $|I_1/(2j\omega)|$.

f) The charge density in each of the triangles T_n^+ and T_n^- is constant over each triangle and the same in each triangle and this is why there is no line charge on edge 1.

g) The normal component of surface current on each side of the edge is the same and this is why there is no line charge on edge 1.

h) Basis function $\boldsymbol{f}_1(\boldsymbol{r})$ has a small but finite component normal to edge 2.

i) If edge 2 is a boundary edge on the scatterer then the tangential component of the surface current will automatically be zero.

j) The charge density in triangle T_1^+ has the same magnitude but opposite sign as the charge density in triangle T_1^- and this is why there is no line charge on edge 1.

19. Consider the recursion formula $y_n + 5y_{n-1} = 1/n$, $y_0 \cong 0.182$ as an approximation to the integral

$$y_n = \int_0^1 \frac{x^n}{x+5} dx$$

The reason this is **not** a good formula is that:

a) it is unstable, the round-off error gets multiplied by 5 at each step.

b) y_0 has been approximated and therefore has round-off error.

c) the truncation error is too high.

20. Consider the use of *orthonormal eigenfunctions* as whole-domain basis functions, $f_n(x)$, in the Galerkin method to find the solution to

$$\frac{d^2}{dx^2} f(x) + kf(x) = g(x),\ 0 < x < 1$$

with $k^2 = \omega^2\mu\varepsilon$ and $g(x)$ given. With orthonormal eigenfunctions we have

$$(f_n, f_m) = \delta_{mn} = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases} \text{ and } \frac{d^2}{dx^2} f_n(x) + kf_n(x) = \lambda_n f_n(x).$$

The coefficients a_n in the approximate solution

$$f = \sum_{n=1}^{N} a_n f_n$$

will be given by

a) $a_m = \dfrac{(f_m, g_m)}{\lambda_m}$

b) $a_m = \dfrac{(f_m, g_m)}{\lambda_m - k}$

c) solving the matrix equation $[l_{mn}][a_n] = [g_m]$ where $l_{mn} = \lambda_n - k$.

d) solving the matrix equation $[l_{mn}][a_n] = [g_m]$ where $l_{mn} = n(n+1)\left[\dfrac{m}{(n+1)(n+m+1)}\right]$

and $g_m = \dfrac{m(3m+8)}{2(m+2)(m+4)}$.

e) none of the above.

21. Consider the 1-D transmission line equations written in conservation-law form:

$$\partial_t \boldsymbol{u}(x,t) + A\partial_x \boldsymbol{u}(x,t) = \mathbf{0}$$

where the solution vector $\boldsymbol{u}$ is given by

$$\boldsymbol{u} = \begin{bmatrix} V \\ I \end{bmatrix}, \text{ and } A = \begin{bmatrix} 0 & e \\ m & 0 \end{bmatrix},\ e = 1/C,\ m = 1/L.$$

and C is the per-unit-length capacitance and L is the per-unit-length inductance of the line. The parameter $v_p^2 = me$ is the phase velocity for waves propagating on the line. If we use the Yee algorithm for this system, it will be stable

a) if we choose $\Delta t > v_p \Delta x$

b) if we choose $\Delta t < v_p \Delta x$

c) only if we choose $\Delta t = v_p \Delta x$

d) if we choose $\Delta t < v_p^2 \Delta x$

e) for all choices of Δt, i.e., the scheme is unconditionally stable.

22. Given a matrix equation $A\boldsymbol{x} = \boldsymbol{b}$ where A is a square matrix, $\boldsymbol{b}$ is a column vector, and $\boldsymbol{x}$ is the column vector we are trying to find, we can write an iterative scheme as

$$M\boldsymbol{x}_{k+1} = (M - A)\boldsymbol{x}_k + \boldsymbol{b}\,,\ \boldsymbol{x}_0 = \boldsymbol{0}\,.$$

Recall that if M is the diagonal part of A we call this iterative scheme the *Jacobi* method, if M is the lower triangular part of A (including the diagonal) we call the iterative scheme the *Gauss-Seidel* method, and if M is the lower triangular part of A with the diagonal elements divided by a parameter $\omega > 1$, called the relaxation parameter, then the iterative scheme is called the *successive overrelaxation* method. Other schemes written as above are possible. Which of the following statements are true:

a) if the spectral radius of A is less than one then the iterative scheme for any M will be stable.

b) if the spectral radius of $B = M^{-1}(M - A)$ is less than one then the iterative scheme for such M that satisfies this equation will be stable.

c) the spectral radius of A and $B = M^{-1}(M - A)$ must both be less than one for the iterative scheme to be stable.

d) the stability of the iterative scheme depends on the initial guess $\boldsymbol{x}_0$.

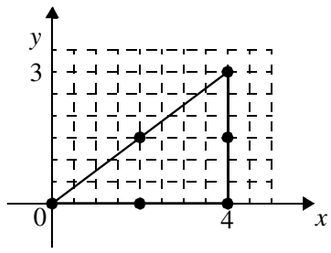
e) the stability of the iterative scheme does not depend on the initial guess $\boldsymbol{x}_0$.

f) the error in the initial guess is given by $\boldsymbol{e}_0 = \boldsymbol{x} - \boldsymbol{x}_0$ and after k iterations the error will be given by $\boldsymbol{e}_k = M^k \boldsymbol{e}_0$.

g) the error in the initial guess is given by $\boldsymbol{e}_0 = \boldsymbol{x} - \boldsymbol{x}_0$ and after k iterations the error will be given by $\boldsymbol{e}_k = A^k \boldsymbol{e}_0$.

h) the error in the initial guess is given by $\boldsymbol{e}_0 = \boldsymbol{x} - \boldsymbol{x}_0$ and after k iterations the error will be given by $\boldsymbol{e}_k = B^k \boldsymbol{e}_0$.

23. Given the second-order triangular element with nodes as shown what is an appropriate second order polynomial basis function over this element such that the basis function is equal to 1 at the node located at $x = 4$, $y = 3$ and zero at all remaining nodes.



a) $\alpha(x, y) = a + bx + cy + dx^2 + exy + fy^2$ with

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} 1 & 4 & 3 & 16 & 12 & 9 \\ 1 & 4 & 3/2 & 16 & 6 & 9/4 \\ 1 & 4 & 0 & 16 & 0 & 0 \\ 1 & 2 & 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3/2 & 4 & 3 & 9/4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

b) $\alpha(x, y) = a + bx + cy + dx^2 + exy + fy^2$ with

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} 1 & 4 & 3 & 16 & 12 & 9 \\ 1 & 4 & 3/2 & 16 & 6 & 9/4 \\ 1 & 4 & 0 & 16 & 0 & 0 \\ 1 & 2 & 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3/2 & 4 & 3 & 9/4 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

c) $\alpha(x, y) = a + bx + cy + dx^2 + exy + fy^2$ with

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & 4 & 4 & 2 & 0 & 2 \\ 3 & 3/2 & 0 & 0 & 0 & 3/2 \\ 16 & 16 & 16 & 4 & 0 & 4 \\ 12 & 6 & 0 & 0 & 0 & 3 \\ 9 & 9/4 & 0 & 0 & 0 & 9/4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

d) $\alpha(x, y) = a + bx + cy + dx^2 + exy + fy^2$ with

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & 4 & 4 & 2 & 0 & 2 \\ 3 & 3/2 & 0 & 0 & 0 & 3/2 \\ 16 & 16 & 16 & 4 & 0 & 4 \\ 12 & 6 & 0 & 0 & 0 & 3 \\ 9 & 9/4 & 0 & 0 & 0 & 9/4 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

e) $\alpha(x, y) = 2L_1^2 - L_1$ with $L_1(x, y)$ the area coordinate for the node located at $x = 0$, $y = 0$.

f) $\alpha(x, y) = 4L_1L_2$ with $L_1(x, y)$ the area coordinate for the node located at $x = 0$, $y = 0$ and $L_2(x, y)$ the area coordinate for the node located at $x = 4$, $y = 3$.

g) $\alpha(x, y) = 4L_1L_2$ with $L_1(x, y)$ the area coordinate for the node located at $x = 4$, $y = 3$ and $L_2(x, y)$ the area coordinate for the node located at $x = 2$, $y = 0$.

h) $\alpha(x, y) = a + bx + cy + dx^2 + exy + fy^2$ with

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} 1 & 4 & 3 & 16 & 12 & 9 \\ 1 & 4 & 3/2 & 16 & 6 & 9/4 \\ 1 & 4 & 0 & 16 & 0 & 0 \\ 1 & 2 & 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3/2 & 4 & 3 & 9/4 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

i) $\alpha(x, y) = a + bx + cy + dx^2 + exy + fy^2$ with

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3/2 & 4 & 3 & 9/4 \\ 1 & 4 & 3/2 & 16 & 6 & 9/4 \\ 1 & 4 & 0 & 16 & 0 & 0 \\ 1 & 2 & 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 4 & 3 & 16 & 12 & 9 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

24. Which of the following statements regarding the finite difference method are true:

a) the stability of a finite difference approximation of Maxwell's equations in the time-domain can be determined using the von Neumann method without regard to the way the boundary conditions are approximated.

b) von Neumann's method can be used to evaluate the stability of the finite difference approximation of the boundary conditions.

c) von Neumann's method tells us that the leap-frog scheme is always stable.

d) von Neumann's method cannot be applied to the leap-frog scheme because it is a two-step scheme.

e) von Neumann's method only gives us the amplification matrix for the scheme and tells us nothing about its stability.

25. In the scattered field formulation of FDTD applied to perfect electric conducting shields in free space:

a) the incident plane wave is modelled using electric surface currents on the shield.

b) the incident plane wave is modelled using magnetic surface currents on the shield.

c) the incident plane wave is modelled by imposing the tangential electric field on the shield.

d) the incident plane wave is modelled by imposing the tangential magnetic field on the shield.